Geometric Numerical Integration of Stochastic Oscillators

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(joint work with D. Cohen and M. J. Grote)

In solving stochastic wave equations numerically, stochastic oscillators play an important role. The pseudo-spectral semi-discretization in space of the considered partial differential equation leads to a large system of coupled stochastic oscillators. We could just use an Euler-Maruyama method to solve the system, but since the oscillators can be highly oscillatory, the time restrictions would be very stringent. We consider a one-dimensional problem which describes a stochastic oscillator. The exact solution of this second-order stochastic differential equation satisfies some properties such as the linear growth of the energy or the second moment.
In this talk, we propose a geometric integrator which is based on the variation-of-constants formula and which solves the equation efficiently. In the linear case it reproduces the above mentioned properties exactly. To solve the nonlinear problem we adapt the trigonometric methods (cf. [1]) to the stochastic setting. The new methods preserve the linear growth of the energy up to a small error.

References

[1] E. Hairer, C. Lubich, G. Wanner, *Geometric numerical integration*, Springer Berlin, 2002.

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