An Adaptive Finite Element Method for Distributed Heat Flux Reconstruction

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Abstract

Based on a posteriori error estimates, we propose an adaptive finite element method for a distributed heat flux reconstruction in a stationary heat conductive system, namely recovering the unknown distributed flux on some inaccessible boundary using partial measurement data on other accessible boundaries. A posteriori error estimates are first derived. Efficiency of the derived error estimator is addressed by showing that the error estimator provides upper and lower bounds on the discretization errors of quantities of interest, up to some constants. It is revealed for the first time that the constant of the upper bound depends explicitly on the regularization parameter, which could be essential for employing adaptive techniques to inverse problems. Numerical experiments are presented to show the applicability and efficiency of the proposed adaptive method based on the derived error estimator.

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