DG Treatment of Sliding Interfaces in 3D Eddy Current Problems

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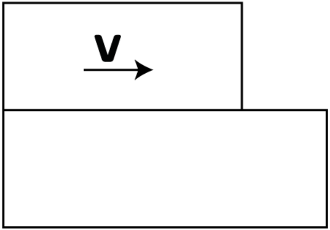
Overview

Why Sliding Interfaces ?

- Eddy Current model for simulation of electrical machines.
- Moving meshes for moving subdomains:



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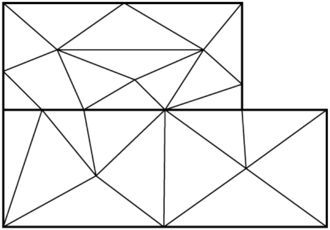
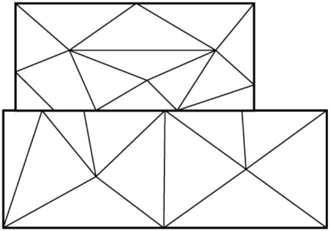
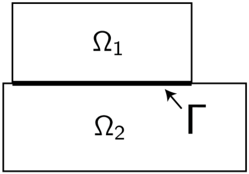


Why Sliding Interfaces ?

- Eddy Current model for simulation of electrical machines.
- Moving meshes for moving subdomains:
  - Conforming meshes become non-conforming: *Sliding Interface*
  - Remeshing unnecessary
  - No (convective) $\sigma \mathbf{v} \times \mathbf{B}$ terms



Temporal Gauged Eddy Current Equations



$$\mathbf{B} = \mathbf{curl}\,\mathbf{A}$$
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$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$$

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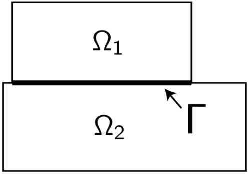

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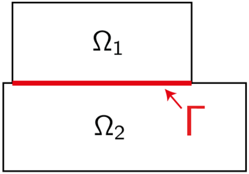

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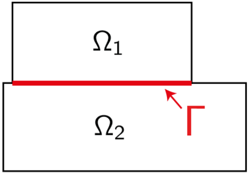

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Properties of our scheme

- Interior Penalty / Nitsche's Method (cf. Stenberg 1998)
  - Sparse and symmetric positive definite matrix
    $\Rightarrow$ Conjugate Gradient (CG)
  - 3D Edge elements in 'interior' of Ω_1, Ω_2.
  - σ and μ can jump over Γ
  - Simple and efficient implementation on Computer
  - Robust w.r.t. mesh motion

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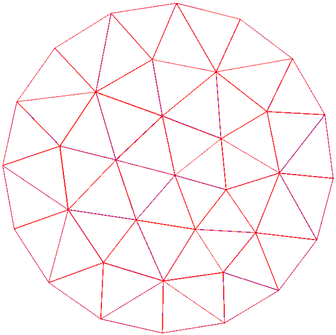

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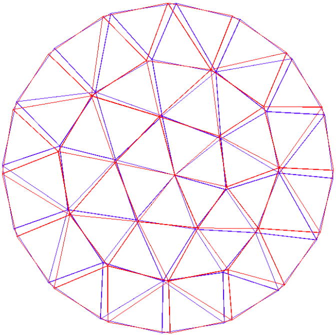

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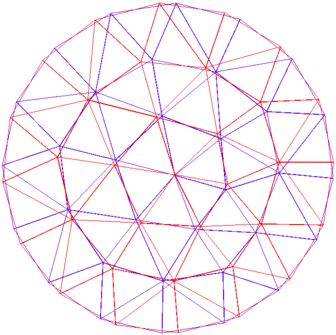

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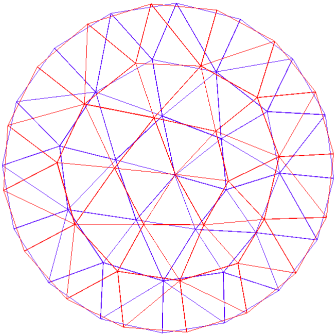

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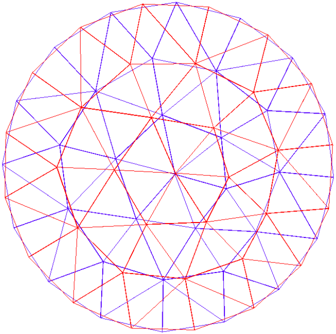

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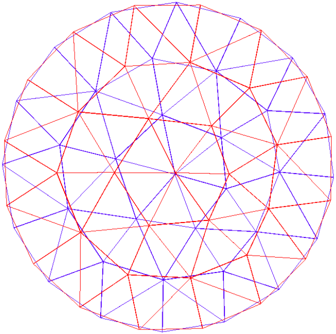

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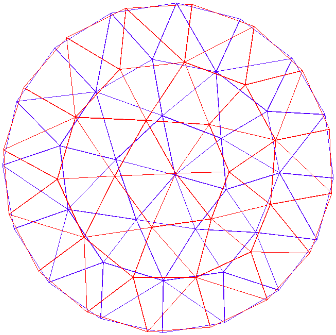

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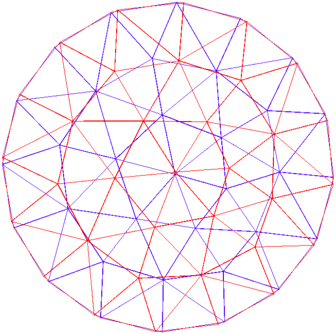

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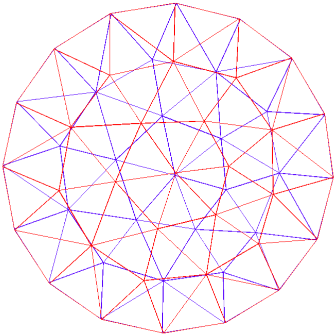

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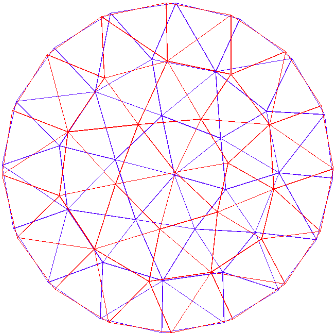

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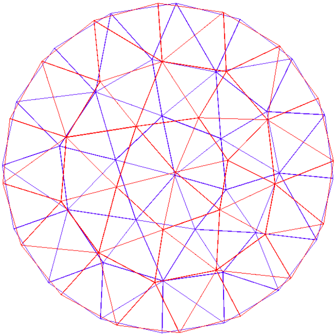

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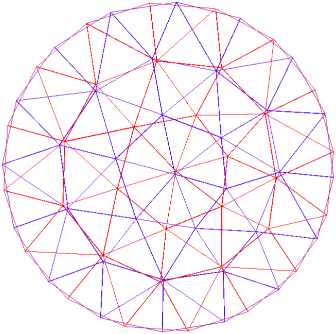

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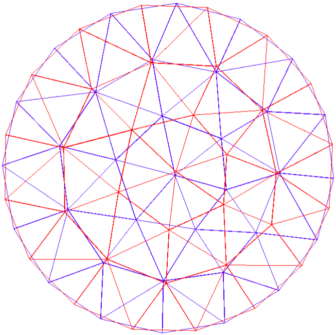

Survey of coupling techniques

- Additional Lagrange multipliers to enforce continuity (Belgacem 1999)
  - Saddle point problem
- Incorporate continuity requirement into approximation space (Rapetti et al. 2002)
  - Invert matrix in every timestep.
- Primal-Dual Coupling across interface(Rodriguez, Hiptmair, and Valli 2005)
  - Block skew symmetric system matrix.
- Locally Discontinuous Galerkin (LDG) Alotto et al. 2002
  - Larger stencil, i.e. system matrix is less sparse.
  - Extension to 3D Edge functions?

Overview

Curl-Curl Problem

- Study model problem:

$$\cancel{\sigma \frac{\partial \mathbf{A}}{\partial t}} + \mathbf{curl}\, \mu^{-1}\, \mathbf{curl}\, \mathbf{A} = \mathbf{j}^i \qquad (1)$$

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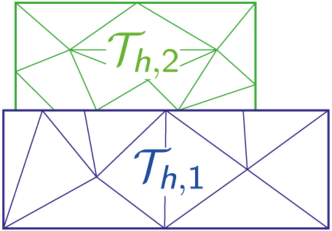
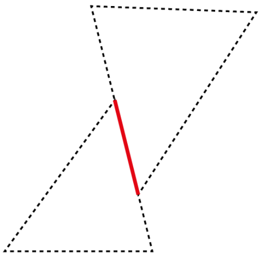


$$\mathcal{T}_h := \mathcal{T}_{h,1} \cup \mathcal{T}_{h,2}$$
$$\mathcal{F}_h = \text{set of all faces of } \mathcal{T}_h$$

Jump and Average operator



- Interior face $F \in \mathcal{F}_h$:

$$[\mathbf{A}]_T := \mathbf{n}_F \times (\mathbf{A}_1 - \mathbf{A}_2) \quad \text{(tangential jump)}$$
$$\{\mathbf{A}\}_\omega := \omega_1 \mathbf{A}_1 + \omega_2 \mathbf{A}_2 \quad \text{(weighted average)}$$

$\omega_1 := \mu_1 / (\mu_1 + \mu_2)$, $\omega_2 := 1 - \omega_1$

- Boundary face $F = \mathcal{F}_h$:

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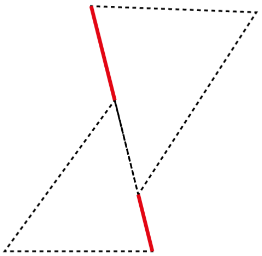

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Symmetric Weighted Interior Penalty (SWIP)

Find $\mathbf{A}_h \in V_h$ such that

$$a_h^{\mathrm{SWIP}}(\mathbf{A}_h, \mathbf{A}'_h) + \varepsilon \int_\Omega \mathbf{A}_h \cdot \mathbf{A}'_h = \int_\Omega \mathbf{j}^i \cdot \mathbf{A}'_h \qquad \text{for all } \mathbf{A}'_h \in V_h \tag{2}$$

where

$$a_h^{\mathrm{SWIP}}(\mathbf{A}, \mathbf{A}') := \int_\Omega \mu^{-1} \,\mathbf{curl}\,\mathbf{A} \cdot \mathbf{curl}\,\mathbf{A}' - \sum_{F \in \mathcal{F}_h} \int_F \left\{\mu^{-1}\,\mathbf{curl}\,\mathbf{A}\right\}_\omega \cdot \left[\mathbf{A}'\right]_T$$
$$- \sum_{F \in \mathcal{F}_h} \int_F \left\{\mu^{-1}\,\mathbf{curl}\,\mathbf{A}'\right\}_\omega \cdot [\mathbf{A}]_T + \sum_{F \in \mathcal{F}_h} \frac{\eta \gamma_{\mu,F}}{h_F} \int_F [\mathbf{A}]_T \cdot \left[\mathbf{A}'\right]_T$$

$$\gamma_{\mu,F} := 2/\left(\mu_1 + \mu_2\right)$$

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Note: If test/trial functions are tangentially continuous functions, the last three terms vanish.

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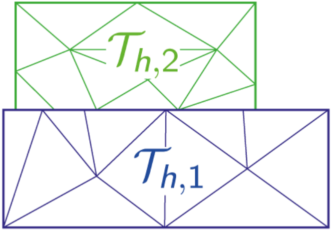
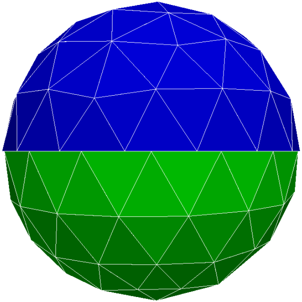
Approximation space V_h



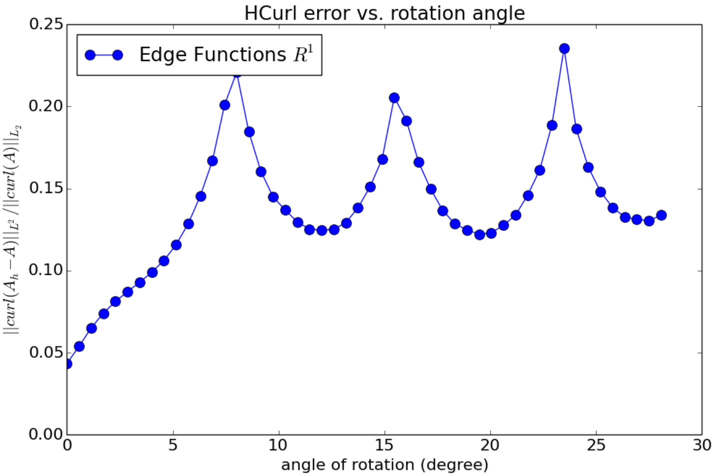
- Edge Functions on $\mathcal{T}_{h,1}$ and $\mathcal{T}_{h,2}$.

Setup of numerical experiment

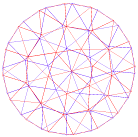


- Ω_1, Ω_2 are two half spheres
- Prescribe analytic solution $(\sin(y), \cos(z), \sin(x))^T$
- Matching dirichlet boundary conditions and right hand side.

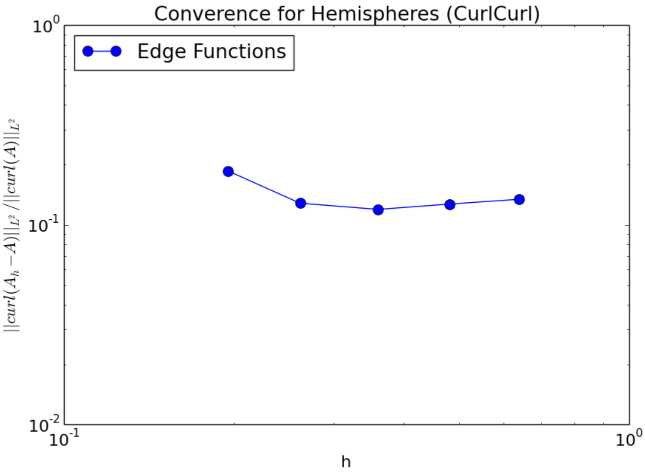
Relative $\mathcal{H}(\mathbf{curl})$ error vs. rotation angle



Relative $\mathcal{H}(\mathbf{curl})$ error vs. h



$\theta = 5.72\,^\circ$



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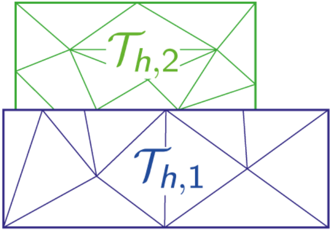

- Edge Functions on $\mathcal{T}_{h,1}$ and $\mathcal{T}_{h,2}$
- Piecewise Polynomials on $\mathcal{T}_h$: $V_h = \mathcal{P}^1[\mathcal{T}_h]^3$
  - Discontinuous across element boundaries

Theorem (A priori error estimate)

Let $\mathbf{A} \in V^* := \mathcal{H}(\mathbf{curl}, \Omega) \cap H^2(P_\Omega)$ *be a solution of the strong formulation* (1), *and let* $\mathbf{A}_h \in V_h \subseteq \mathcal{P}^1[\mathcal{T}_h]^3$ *solve the variational formulation* (2). *Furthermore assume that* η *is sufficiently large. Then there is* C *independent of* h *such that*

$$\|\mathbf{A} - \mathbf{A_h}\|_{SWIP} < C \inf_{\mathbf{v}_h \in V_h} \|\mathbf{A} - \mathbf{v}_h\|_{SWIP,*},$$

and the discrete problem (2) *is well-posed.*

Definition

$$\|\mathbf{A}\|^2_{\text{SWIP}} = \left\|\mu^{-1/2}\,\mathbf{curl}\,\mathbf{A}\right\|^2_{L^2(\Omega)} + \left\|\varepsilon^{1/2}\mathbf{A}\right\|^2_{L^2(\Omega)} + \sum_{F \in \mathcal{F}_h} \frac{\gamma_{\mu,F}}{h_F} \|[\mathbf{A}]_T\|^2_{L^2(F)}$$

$$\|\mathbf{A}\|^2_{\text{SWIP},\,*} = \|\mathbf{A}\|^2_{\text{SWIP}} + \sum_{T \in \mathcal{T}_h} h_T \left\|\mu^{-1/2}\,\mathbf{curl}\,\mathbf{A}\Big|_T\right\|^2_{L^2(\partial T)}$$

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$$\|\mathbf{A}\|^2_{\mathrm{SWIP},\,*} = \|\mathbf{A}\|^2_{\mathrm{SWIP}} + \sum_{T\in\mathcal{T}_h} h_T \left\|\mu^{-1/2}\,\mathbf{curl}\,\mathbf{A}\Big|_T\right\|^2_{L^2(\partial T)}$$

Approximation space V_h

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- first order Edge Functions R^1
  - Monk 2003[Lemma 5.52]

$$\exists r_h : V^* \mapsto V^h \text{ s.t. } \|\mathbf{n}_T \times (\mathbf{A} - r_h(\mathbf{A}))\|_{L^2(F)} < Ch^{1/2}\|A\|_{\mathcal{H}^2(T)}$$
$$\forall T \in \mathcal{T}_h, F \in \mathcal{F}_T$$

  - Upper bound for approximation error: $\mathcal{O}(h^0)$

Approximation space V_h

Definition

$$\|\mathbf{A}\|_{\text{SWIP}}^2 = \left\|\mu^{-1/2}\,\mathbf{curl}\,\mathbf{A}\right\|_{L^2(\Omega)}^2 + \left\|\varepsilon^{1/2}\mathbf{A}\right\|_{L^2(\Omega)}^2 + \sum_{F\in\mathcal{F}_h} \frac{\gamma_{\mu,F}}{h_F}\,\|[\mathbf{A}]_T\|_{L^2(F)}^2$$

$$\|\mathbf{A}\|_{\text{SWIP},*}^2 = \|\mathbf{A}\|_{\text{SWIP}}^2 + \sum_{T\in\mathcal{T}_h} h_T \left\|\mu^{-1/2}\,\mathbf{curl}\,\mathbf{A}\Big|_T\right\|_{L^2(\partial T)}^2$$

- Piecewise Polynomials $\mathcal{P}^1[\mathcal{T}_h]^3$
  - Easily prove:

$$\exists \pi_h : V^* \mapsto V_h \text{ s.t. } \|\mathbf{A} - \pi_h(\mathbf{A})\|_{\text{SWIP},*} = \mathcal{O}(h^1)$$

  - Upper bound $\mathcal{O}(h^1)$ *independent of mesh position*

Remarks

- First order edge functions R_1:
  - *fail* at the sliding interface
  - *fail* "generally" at boundary if *inhomogeneous Dirichlet conditions* are *weakly* enforced.
  - *work* for conforming mesh: jump terms drop out!
- $\mathcal{P}^1[\mathcal{T}_h]^3$ work but is expensive (10x dofs)

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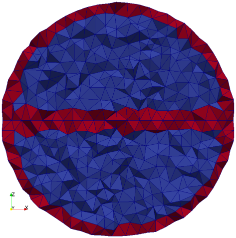
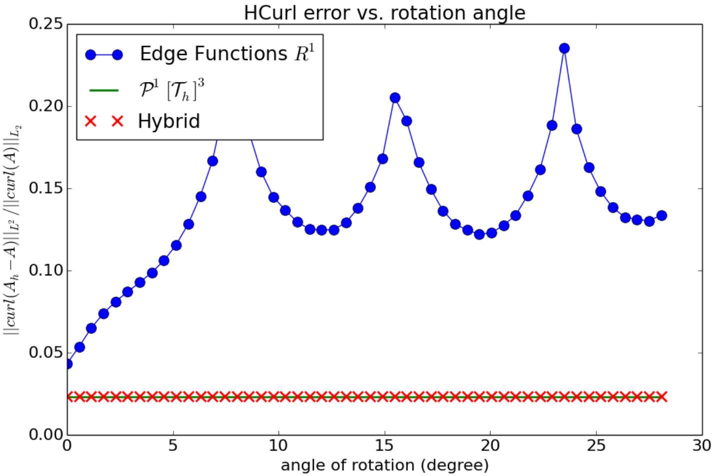
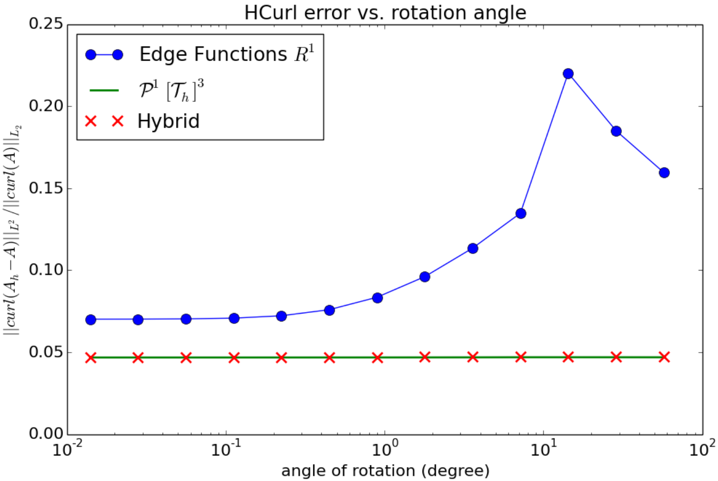
- $\mathcal{P}^1[\mathcal{T}_h]^3$ work but is expensive (10x dofs)
- Natural Idea: Combine the two approaches



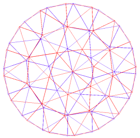
Relative $\mathcal{H}(\mathbf{curl})$ error vs. rotation angle



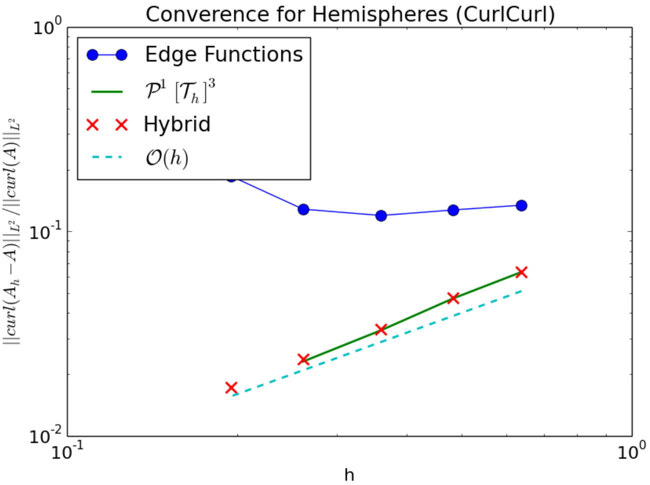
Relative $\mathcal{H}(\mathbf{curl})$ error vs. rotation angle



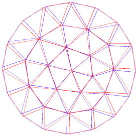
Relative $\mathcal{H}(\mathbf{curl})$ error vs. h



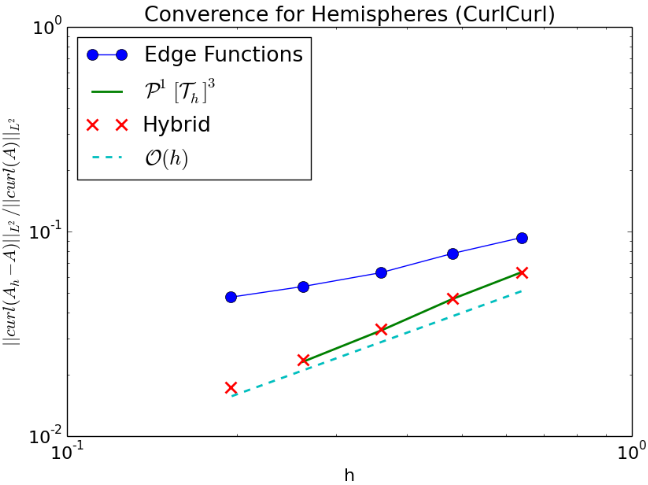
$\theta = 5.72\,^\circ$



Relative $\mathcal{H}(\mathbf{curl})$ error vs. h



$\theta = 0.57\,^\circ$

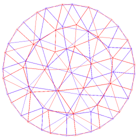


Overview

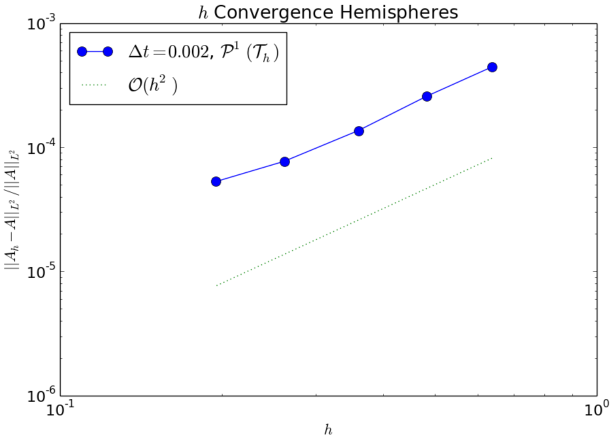
Eddy Current

- Implicit Euler timestepping
- So far: No motion but non-conforming interface

L^2 Convergence for Eddy Current



$\theta = 28.6^\circ$



Summary

- Nitsche's Method doesn't work with first order Edge Functions.
- Discontinuous Polynomials $\mathcal{P}^1[\mathcal{T}_h]^3$ work robustly but are expensive.
- Hybrid approach successful, little overhead and robust.
- Proof of Convergence for Curl-Curl Problem.
- Extension to Eddy Current works.

Questions ???

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